

Spin polarized transport in the weak link between f-wave superconductors

Gholamreza Rashedi¹, Yuri A. Kolesnichenko²

¹ *Institute for Advanced Studies in Basic Sciences, 45195-159, Zanjan, Iran*

² *B.Verkin Institute for Low Temperature Physics Engineering of National Academy of Sciences of Ukraine, 47, Lenin ave , 61103, Kharkov, Ukraine*

(Dated: March 2, 2009)

The spin current in the Josephson junction as a weak-link (interface) between misorientated triplet superconductors is investigated theoretically for the models of the order parameter in UPt_3 . Green functions of the system are obtained from the quasiclassical Eilenberger equations. The analytical results for the charge and spin currents are illustrated by numerical calculations for the certain misorientation angles of gap vector of superconductors. As the main result of this paper, it is found that, at some values of the phase difference, at which the charge current is exactly zero, the spin current has its maximum value. Furthermore, it is shown that the origin of spin current is the misorientation between gap vectors of triplet superconductors.

PACS numbers: 74.50.+r, 74.70.Pq, 74.70.Tx, 72.25.-b

I. INTRODUCTION

Triplet superconductivity has become one of the most interesting topics of condensed matter physics^{1,2}, particularly in view of the recently discovered ferromagnetic superconductivity^{3,4}. The mechanism of pairing, physics of interaction and gap structure in this type of superconductors have been the subject of many experimental and theoretical works^{5,6,7,8,9,10,11}. The Cooper pairing in the triplet superconductors has been investigated, for example, using the thermal conductivity in Refs.^{12,13} and Knight shift experiments in Refs.^{14,15}. Also, the Josephson effect in the point contact between triplet superconductors has been studied in Ref.¹⁶. These weak-link structures have been used to demonstrate the order parameter symmetry in Ref.¹⁷. Eventually, the f -wave symmetry of order parameter has been proposed for UPt_3 and Sr_2RuO_4 compounds. In addition, the spin polarized transport through the systems consisting of superconductors, normal metals, ferromagnetic layers and other structures as one of the modern topics of mesoscopic physics, has attracted much attention recently^{18,19,20,21}. In the present paper, the ballistic Josephson weak-link as the interface between two bulk of f -wave superconductors with different orientations of the crystallographic axes has been investigated. It is shown that the current-phase dependencies are totally different from the current-phase dependencies of the junction between conventional (s -wave) superconductors^{22,23} and high T_c (d -wave) superconductors²⁴. It is found that for the certain values of the misorientation, the spin-current in the both directions, tangential and perpendicular to the interface, may exist and it has totally unusual dependence on the external phase difference. The effect of misorientation on the spin current is investigated. It is observed that the misorientation between gap vectors is the origin of the spin current. As the important result of this paper, it is obtained that, at some of certain values of phase difference, at which the charge current is zero, the spin current has the finite value. Another result of

the paper is the capability of this proposed experiment for polarization of the spin transport using the junction between f -wave superconductors. Eventually, one of the states and geometries of our system can be used as a switch which is able to divide the spin and charge currents into two parts: parallel and perpendicular to the interface.

The organization of the rest of this paper is as follows. In Sec.II we describe our configuration, which is investigated. For a non-self-consistent model of the order parameter, the quasiclassical Eilenberger equations³¹ are solved and suitable Green functions are obtained analytically. In Sec. III the obtained formulas for the Green functions are used for calculation the charge and spin current densities at the interface. An analysis of numerical results will be done in Sec.IV. The paper will be finished with some conclusions in Sec.V.

II. FORMALISM AND BASIC EQUATIONS

We consider a model of a flat interface $y = 0$ between two misorientated f -wave superconducting half-spaces (Fig.1) as a ballistic Josephson junction. In the quasiclassical ballistic approach, in order to calculate the charge and spin current, we use “transport-like” equations³¹ for the energy integrated Green functions $\check{g}(\hat{\mathbf{v}}_F, \mathbf{r}, \varepsilon_m)$

$$\mathbf{v}_F \nabla \check{g} + [\varepsilon_m \check{\sigma}_3 + i\check{\Delta}, \check{g}] = 0, \quad (1)$$

and the normalization condition

$$\check{g}\check{g} = \check{1}, \quad (2)$$

where $\varepsilon_m = \pi T(2m + 1)$ are discrete Matsubara energies $m = 0, 1, 2, \dots$, T is the temperature and \mathbf{v}_F is the Fermi velocity and $\check{\sigma}_3 = \hat{\sigma}_3 \otimes \hat{I}$ in which $\hat{\sigma}_j$ ($j = 1, 2, 3$) are Pauli matrices. The Matsubara propagator \check{g} can be written in the standard form:

$$\check{g} = \begin{pmatrix} g_1 + \mathbf{g}_1 \hat{\sigma} & (g_2 + \mathbf{g}_2 \hat{\sigma}) i \hat{\sigma}_2 \\ i \hat{\sigma}_2 (g_3 + \mathbf{g}_3 \hat{\sigma}) & g_4 - \hat{\sigma}_2 \mathbf{g}_4 \hat{\sigma} \hat{\sigma}_2 \end{pmatrix}, \quad (3)$$

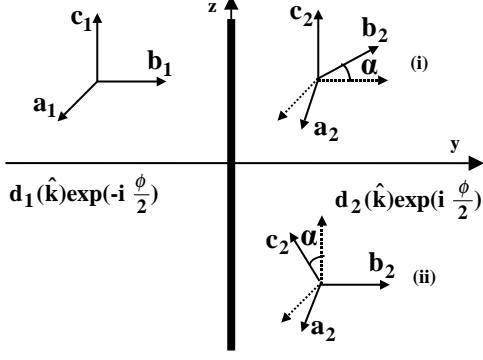


FIG. 1: Scheme of a flat interface between two superconducting bulks, which are misorientated as much as α .

where, the matrix structure of the off-diagonal self energy $\check{\Delta}$ in the Nambu space is

$$\check{\Delta} = \begin{pmatrix} 0 & \mathbf{d}\hat{\sigma}i\hat{\sigma}_2 \\ i\hat{\sigma}_2\mathbf{d}^*\hat{\sigma} & 0 \end{pmatrix}. \quad (4)$$

In this paper, the unitary states, for which $\mathbf{d} \times \mathbf{d}^* = 0$, is investigated. Also, the unitary states vectors $\mathbf{d}_{1,2}$ can be written as

$$\mathbf{d}_n = \Delta_n \exp i\psi_n, \quad (5)$$

where $\Delta_{1,2}$ are the real vectors in the left and right sides of the junction. The gap (order parameter) vector \mathbf{d} has to be determined from the self-consistency equation:

$$\mathbf{d}(\hat{\mathbf{v}}_F, \mathbf{r}) = \pi T N(0) \sum_m \langle V(\hat{\mathbf{v}}_F, \hat{\mathbf{v}}'_F) \mathbf{g}_2(\hat{\mathbf{v}}'_F, \mathbf{r}, \varepsilon_m) \rangle \quad (6)$$

where $V(\hat{\mathbf{v}}_F, \hat{\mathbf{v}}'_F)$, is a potential of pairing interaction, $\langle \dots \rangle$ stands for averaging over the directions of an electron momentum on the Fermi surface $\hat{\mathbf{v}}'_F$ and $N(0)$ is the electron density of states at the Fermi level of energy. Solutions to Eqs. (1) and (6) must satisfy the conditions for Green functions and vector \mathbf{d} in the bulks of the superconductors far from the interface as follow:

$$\check{g}(\pm\infty) = \frac{\varepsilon_m \check{\sigma}_3 + i\check{\Delta}_{2,1}}{\sqrt{\varepsilon_m^2 + |\mathbf{d}_{2,1}|^2}}; \quad (7)$$

$$\mathbf{d}(\pm\infty) = \mathbf{d}_{2,1}(\hat{\mathbf{v}}_F) \exp\left(\mp \frac{i\phi}{2}\right), \quad (8)$$

where ϕ is the external phase difference between the order parameters of the bulks. Eqs. (1) and (6) have to be supplemented by the continuity conditions at the interface between superconductors. For all quasiparticle trajectories, the Green functions satisfy the boundary conditions both in the right and left bulks as well as at the interface.

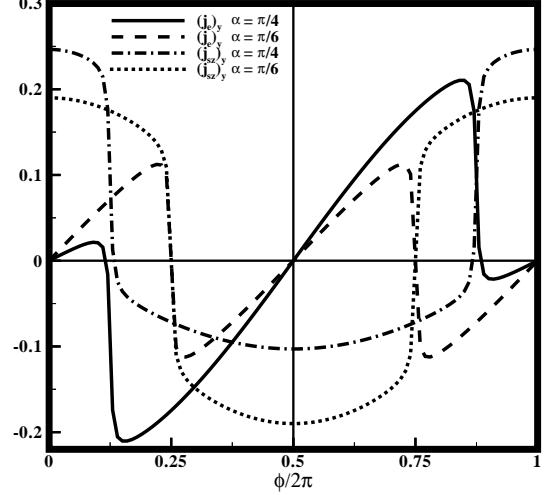


FIG. 2: Charge and spin current (s_z) versus the phase difference ϕ for the planar state (16), geometry (i) and the different misorientations. Currents are given in units of $j_{0,e} = \frac{\pi}{2}eN(0)v_F\Delta_0(0)$ and $j_{0,s} = \frac{\pi}{4}\hbar N(0)v_F\Delta_0(0)$ respectively.

The system of equations (1) and (6) can be solved only numerically. For unconventional superconductors such solution requires the information of the function $V(\hat{\mathbf{v}}_F, \hat{\mathbf{v}}'_F)$. This information, as that of the nature of unconventional superconductivity in novel compounds, in most cases is unknown. Usually, the spatial variation of the order parameter and its dependence on the momentum direction can be separated in the form of $\Delta(\hat{\mathbf{v}}_F, y) = \Delta(\hat{\mathbf{v}}_F)\Psi(y)$. It has been shown that the absolute value of a self-consistent order parameter and $\Psi(y)$ are suppressed near the interface and at the distances of the order of the coherence length, while its dependence on the direction in the momentum space ($\Delta(\hat{\mathbf{v}}_F)$) remains unaltered²⁵. Consequently, this suppression doesn't influence the Josephson effect drastically. This suppression of the order parameter keeps the current-phase dependence unchanged but, it changes the amplitude value of the current. For example, it has been verified in Refs.²⁴ for the junction between unconventional d -wave, in Ref.²⁵ for the case of “ f -wave” superconductors and in Ref.²⁶ for pinholes in ^3He that, there is a good qualitative agreement between self-consistent and non-self-consistent results. Also, it has been observed that the results of the non-self-consistent investigation of $D-N-D$ structure in the paper²⁷ are coincident with the experimental results of the paper²⁸ and the results of the non-self-consistent model in paper²⁹ are similar to the superfluid weak-link experiment³⁰. Note that any self-consistent solution can be found for some models of pairing potential and the certain values of parameters. Consequently, despite the fact that this estimation cannot be applied directly for a quantitative analyze of the real experiment, only a

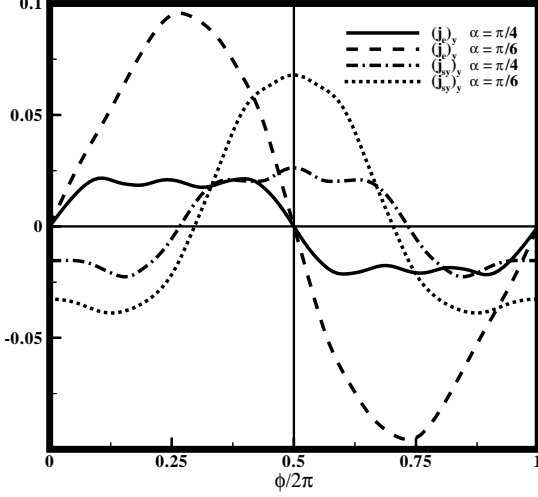


FIG. 3: Charge and spin current (s_y) versus the phase difference ϕ for the axial state (15), geometry (ii) and the different misorientations (y -component).

qualitative comparison of calculated and experimental current-phase relations is possible. In our calculations, a simple model of the constant order parameter up to the interface is considered and the pair breaking and the scattering on the interface are ignored. We believe that under these strong assumptions our results describe the real situation qualitatively. In the framework of such model, the analytical expressions for the charge and spin current can be obtained for an arbitrary form of the order parameter.

III. ANALYTICAL RESULTS.

The solution of Eqs. (1) and (6) allows us to calculate the charge and spin current densities. The expression for the charge current is:

$$\mathbf{j}_e(\mathbf{r}) = 2i\pi eTN(0) \sum_m \langle \mathbf{v}_F g_1(\hat{\mathbf{v}}_F, \mathbf{r}, \varepsilon_m) \rangle \quad (9)$$

and for the spin current we have:

$$\mathbf{j}_{s_i}(\mathbf{r}) = 2i\pi\left(\frac{\hbar}{2}\right)TN(0) \sum_m \langle \mathbf{v}_F (\hat{\mathbf{e}}_i \mathbf{g}_1(\hat{\mathbf{v}}_F, \mathbf{r}, \varepsilon_m)) \rangle \quad (10)$$

where, $\hat{\mathbf{e}}_i = (\hat{\mathbf{x}}, \hat{\mathbf{y}}, \hat{\mathbf{z}})$. We assume that the order parameter does not depend on coordinates and in each half-space it equals to its value (8) far from the interface in the left or right bulks. For such a model, the current-phase dependence of a Josephson junction can be calculated analytically. It enables us to analyze the main features of current-phase dependence for the different models of the order parameter of “ f -wave” superconductivity. The

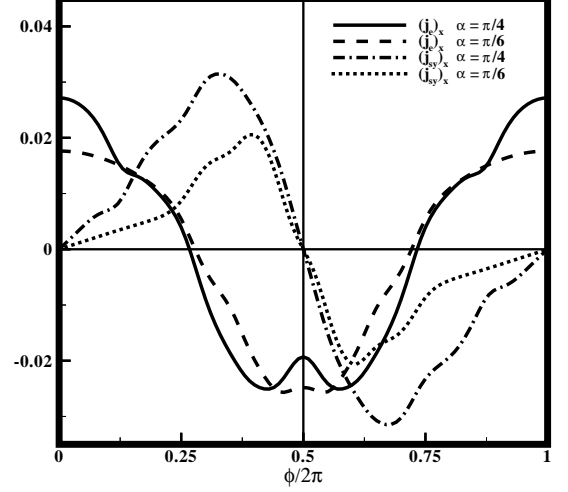


FIG. 4: Charge and spin current (s_y) versus the phase difference ϕ for the axial state (15), geometry (ii) and the different misorientations (x -component)

Eilenberger equations (1) for Green functions \check{g} , which are supplemented by the condition of continuity of solutions across the interface, $y = 0$, and the boundary conditions at the bulks, are solved for a non-self-consistent model of the order parameter analytically. Two diagonal terms of Green matrix which determine the current densities at the interface, $y = 0$, are following. For the relative term to the charge current we obtain:

$$g_1(0) = \frac{\varepsilon_m(\Omega_1 + \Omega_2) \cos \beta + i\eta \sin \beta (\Omega_1 \Omega_2 + \varepsilon_m^2)}{i\eta \sin \beta \varepsilon_m(\Omega_1 + \Omega_2) + \cos \beta (\Omega_1 \Omega_2 + \varepsilon_m^2) + \mathbf{\Delta}_1 \mathbf{\Delta}_2} \quad (11)$$

and for the case of spin current we have:

$$\mathbf{g}_1(0) = \frac{\eta \mathbf{\Delta}_1 \times \mathbf{\Delta}_2}{(A + 2B) |\mathbf{d}_1|^2 |\mathbf{d}_2|^2} [(B-1)^2 \exp(i\beta)(\eta \Omega_1 + \varepsilon_m) \times (\eta \Omega_2 + \varepsilon_m) - (B+1)^2 \exp(-i\beta)(\eta \Omega_2 - \varepsilon_m)(\eta \Omega_1 - \varepsilon_m)] \quad (12)$$

where $\eta = \text{sgn}(v_y)$, $\Omega_n = \sqrt{\varepsilon_m^2 + |\mathbf{d}_n|^2}$, $\beta = \psi_1 - \psi_2 + \phi$,

$$B = \frac{\eta \varepsilon_m(\Omega_1 + \Omega_2) \cos \beta + i \sin \beta (\Omega_1 \Omega_2 + \varepsilon_m^2)}{i\eta \sin \beta \varepsilon_m(\Omega_1 + \Omega_2) + \cos \beta (\Omega_1 \Omega_2 + \varepsilon_m^2) + \mathbf{\Delta}_1 \mathbf{\Delta}_2} \quad (13)$$

and

$$A = \frac{\mathbf{\Delta}_1 \mathbf{\Delta}_2 (B-1) \exp(i\beta)}{(\eta \Omega_1 - \varepsilon_m)(\eta \Omega_2 - \varepsilon_m)} + \frac{\mathbf{\Delta}_1 \mathbf{\Delta}_2 (B+1) \exp(-i\beta)}{(\eta \Omega_1 + \varepsilon_m)(\eta \Omega_2 + \varepsilon_m)} \quad (14)$$

Also, $n = 1, 2$ label the left and right half-spaces respectively. We consider a rotation \check{R} only in the right superconductor (see, Fig.1), (i.e., $\mathbf{d}_2(\hat{\mathbf{k}}) = \check{R} \mathbf{d}_1(\check{R}^{-1} \hat{\mathbf{k}})$, $\hat{\mathbf{k}}$ is

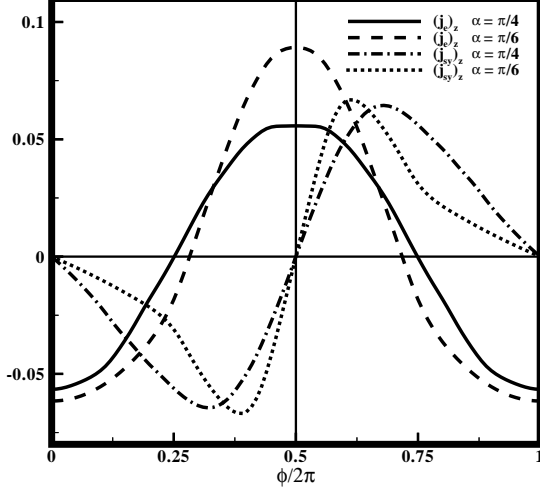


FIG. 5: Charge and spin current (s_y) versus the phase difference ϕ for the axial state (15), geometry (ii) and the different misorientations (z -component)

the unit vector in the momentum space). The crystallographic c -axis in the left half-space is selected parallel to the partition between the superconductors (along z -axis in Fig.1). To illustrate the results obtained by computing the formula (11,12), we plot the current-phase diagrams for the different models of the “ f -wave” pairing symmetry (15,16) and for two different geometries. These two geometries are corresponding to the different orientations of the crystals in the right and left sides of the interface (see, Fig.1):

- (i) The basal ab -plane in the right side has been rotated around the c -axis by α ; $\hat{\mathbf{c}}_1 \parallel \hat{\mathbf{c}}_2$.
- (ii) The c -axis in the right side has been rotated around the axis perpendicular to the interface (y -axis in Fig.1) by α ; $\hat{\mathbf{b}}_1 \parallel \hat{\mathbf{b}}_2$.

Further calculations require a certain model of the gap vector (vector of order parameter) \mathbf{d} .

IV. ANALYSIS OF NUMERICAL RESULTS

In the present paper, two most probable forms of the f -wave order parameter vector in UPt_3 are considered. The first model which is successful to explain the properties of the B -phase of UPt_3 is the axial state. This state describes the strong spin-orbital coupling with vector \mathbf{d} directed along the \mathbf{c} axis of the lattice and it is:

$$\mathbf{d}(T, \mathbf{v}_F) = \Delta_0(T) \hat{\mathbf{z}} k_z (k_x + ik_y)^2. \quad (15)$$

The coordinate axes $\hat{\mathbf{x}}, \hat{\mathbf{y}}, \hat{\mathbf{z}}$ here and below are chosen along the crystallographic axes $\hat{\mathbf{a}}, \hat{\mathbf{b}}, \hat{\mathbf{c}}$ in the left side of Fig.1. The function $\Delta_0 = \Delta_0(T)$ in Eq.(15) and below describes the dependence of the order parameter \mathbf{d} on

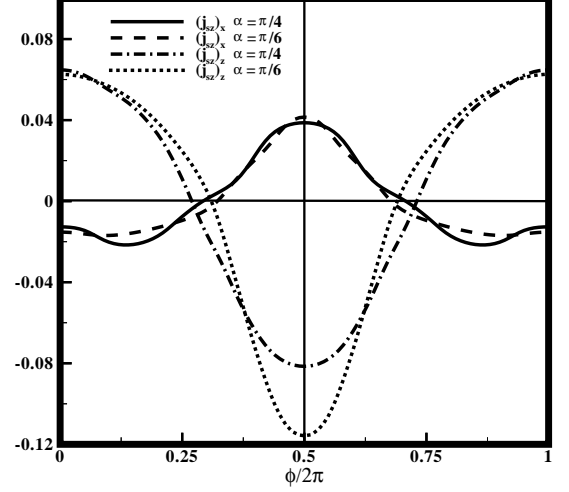


FIG. 6: Tangential spin current (s_z) versus the phase difference ϕ for the planar state (16), geometry (ii) and the different misorientations. The perpendicular component (y -direction) of the spin current is absent.

the temperature T (our numerical calculations have been done at the temperatures close to the $T = 0$). The second model of the order parameter which describes the weak spin-orbital coupling in UPt_3 states, is the unitary planar state. The planar model of gap vector is:

$$\mathbf{d}(T, \mathbf{v}_F) = \Delta_0(T) k_z (\hat{\mathbf{x}} (k_x^2 - k_y^2) + \hat{\mathbf{y}} 2k_x k_y). \quad (16)$$

Using these two models of order parameters (15,16) and solutions to the Eilenberger equations (11,12), we have calculated the spin current and charge current densities at the interface numerically. These numerical results are listed below:

- 1) The spin current can be present, only when misorientation between gap vectors exists. Because in our Green function (12), the spin current is proportional to the “cross product” between the left and right gap vectors. For instance, the spin current for the case of the axial state (15) and geometry (i) is zero, because both of the gap vectors are in the same direction ($\hat{\mathbf{z}}$). (Geometry (i) is a rotation as much as α , around the z axis).

- 2) In Fig.2 it is shown that for the planar state and geometry (i), it is possible to observe the current of s_z in the direction perpendicular to the interface, but in Figs.3,4 and 5, it is demonstrated that, for the axial state and geometry (ii), only the current of s_y can be observed. Consequently, this kind of junction can be applied as a polarizer or filter for the spin currents.

However, for the planar state and geometry (ii), all terms of the spin current (s_x , s_y and s_z) can be observed (see Eq.12).

- 3) In Figs.3-9 (planar states), it is shown that the value of the phase differences in which the currents are in the

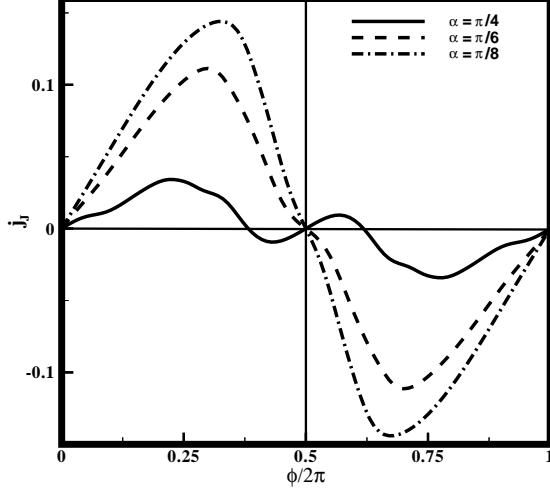


FIG. 7: The perpendicular component of the charge current (Josephson) versus the phase difference ϕ for the planar state (16), geometry (ii) and the different misorientations. The tangential components (x and z -directions) are absent.

maxima, minima and zero values, are not very sensitive to the misorientation angle α , while the amplitude of maxima and minima, are strongly dependent on the value of misorientation α .

4) In the Figs.2,3 and Fig.6, 7, while the charge currents are the odd functions of ϕ with respect to the line of $\phi = \pi$, the spin currents are even functions of the phase difference; $\mathbf{j}_e(\phi = \pi + \delta\phi) = -\mathbf{j}_e(\phi = \pi - \delta\phi)$ and for the spin current $\mathbf{j}_{s_i}(\phi = \pi + \delta\phi) = \mathbf{j}_{s_i}(\phi = \pi - \delta\phi)$. On the contrary, in the Figs.4 and 5, the charge and spin currents are even and odd functions of ϕ with respect to the line of $\phi = \pi$, respectively; $\mathbf{j}_e(\phi = \pi + \delta\phi) = \mathbf{j}_e(\phi = \pi - \delta\phi)$ and $\mathbf{j}_{s_i}(\phi = \pi + \delta\phi) = -\mathbf{j}_{s_i}(\phi = \pi - \delta\phi)$.

5) In Fig.2, the perpendicular component of the spin and charge current in terms of the external phase difference ϕ for the case of the planar state (16), geometry (i) and for two different misorientations are plotted. The solid line is the charge current-phase dependence¹⁶. Also, at the $\phi = 0$, $\phi = \pi$ and $\phi = 2\pi$, the charge current (Josephson current) is zero while the spin current has the finite value.

6) The perpendicular component of the charge (Josephson current) and spin current for the case of the axial state (15) and geometry (ii) are plotted in Fig.3, and the tangential components of them, are plotted in Figs.4,5. The charge current-phase diagrams have been obtained before in paper¹⁶ and they are totally different from the case of conventional superconductors in the paper²². At the phase values of $\phi = 0$, $\phi = \pi$ and $\phi = 2\pi$, in which the charge current is exactly zero, the spin current has the finite values and may select its maximum value. In Figs.2,3 and specially Fig.9, for a small value of misori-

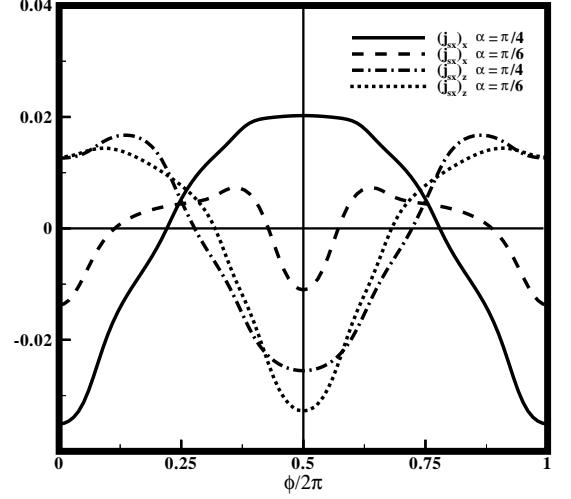


FIG. 8: Tangential spin currents (s_x) versus the phase difference ϕ for the planar state (16), geometry (ii) and the different misorientations. The perpendicular component (y -direction) of the spin current is absent.

entation we have a very long but narrow peak in the spin current phase diagram, close to the $\phi = \pi$.

7) Both the planar state with geometry (i) and the axial state with geometry (ii) can be applied as the filter for polarization of the spin transport (see Figs2-5), the former transports only the s_z but the latter case flows the s_y (see .12). In addition, the planar states with geometry (ii) can be used as a switch for the spin and charge current into two directions: parallel and perpendicular to the interface. In this case, the spin and charge currents select only one of the directions parallel or perpendicular to the interface. Namely, it is impossible to observe the tangential and perpendicular components of the currents at the same time for planar state with geometry (ii)(Figs.6-9).

V. CONCLUSIONS

We have theoretically studied the spin current in the ballistic Josephson junction in the model of an ideal transparent interface between two misorientated f -wave superconductors which are subject to a phase difference ϕ . Our analysis has shown that the misorientation and different models of the gap vectors influence the spin current. This has been shown for the charge current in the paper¹⁶. The misorientation changes strongly the critical values of both the spin current and charge current. It has been obtained that the spin current is the result of the misorientation between the gap vectors. Furthermore, it is observed that the different models of the gap vectors and geometries can be applied to the polarization of the

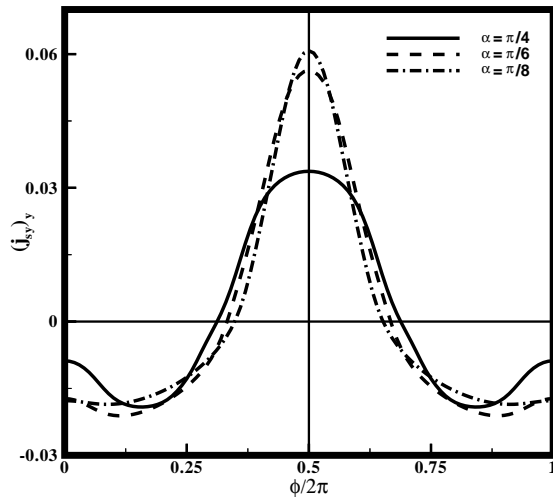


FIG. 9: Perpendicular component of the spin current (s_y) versus the phase difference ϕ for the planar state (16), geometry (ii) and the different misorientations. The tangential components (x and z directions) are absent.

spin transport. Another result of these calculations is the state in which the currents select one of the two possible directions (perpendicular and parallel to the interface)

to flow. This property can be used as a switch to control the direction of the charge and spin current. Finally, as an interesting and new result, it is observed that at some certain values of the phase difference ϕ , the charge-current vanishes while the spin-current flows, although the carriers of both spin and charge are the same (electrons). The spatial variation of the phase of the order parameter plays a role as the origin of the charge current and, similarly, due to the broken $G^{\text{spin-orbit}}$ symmetry, a spatial difference of the gap vectors in two half-spaces causes spin currents. This is because there is a position-dependent phase difference between “spin up” and “spin down” Cooper pairs and, although the total charge current vanishes, there can be a net transfer of the spin. Therefore, in our system, there is a discontinuous jump between the gap vectors and, consequently the spin currents should generally be present. For instance, if up-spin states and down-spin states have a velocity in the opposite direction, the charge currents cancel each other whereas the spin current is being transported. Mathematically speaking, $\mathbf{j}_{\text{charge}} = \mathbf{j}_{\uparrow} + \mathbf{j}_{\downarrow}$, $\mathbf{j}_{\text{spin}} = \mathbf{j}_{\uparrow} - \mathbf{j}_{\downarrow}$, so it is possible to find the state in which one of these current terms is zero and the other term has the finite value²¹. In addition the spin imbalance which is the result of the different density of states for “spin-up” and “spin-down” can be the other reason of spin current²⁰. In conclusion, the spin current in the absence of the charge current can be observed.

- ¹ Y. Maeno, H. Hashimoto, K. Yoshida, S. Nashizaki, T. Fujita, J.G. Bednorz, and F. Lichtenberg, *Nature*, **372**, 532 (1994).
- ² A.P. Mackenzie and Y. Maeno, *Rev. Mod. Phys.* **75**, 657 (2003).
- ³ V.P. Mineev and T. Champel, *Phys. Rev. B* **69**, 144521 (2004).
- ⁴ M.B. Walker and K.V. Samokhin, *Phys. Rev. Lett.* **88**, 207001 (2002).
- ⁵ T. Takimoto, *Phys. Rev. B* **62**, R14641 (2000).
- ⁶ K. Ishida, H. Mukuda, Y. Kitaoka, Z.Q. Mao, Y. Mori, and Y. Maeno, *Phys. Rev. Lett.* **84**, 5387 (2000).
- ⁷ Y. Asano, Y. Tanaka, Y. Matsuda, and S. Kashiwaya, *Phys. Rev. B* **68**, 184506 (2003).
- ⁸ N. Karchev, *Phys. Rev. B* **67**, 054416 (2003).
- ⁹ K. Deguchi, Z.Q. Mao, H. Yaguchi, and Y. Maeno, *Phys. Rev. Lett.* **92**, 047002 (2004).
- ¹⁰ E.I. Blount, *Phys. Rev. B* **32**, 2935 (1985).
- ¹¹ P.W. Anderson, *Phys. Rev. B* **30**, 4000 (1984).
- ¹² K. Izawa, H. Takahashi, H. Yamaguchi, Y. Matsuda, M. Suzuki, T. Sasaki, T. Fukase, Y. Yoshida, R. Settai, and Y. Onuki, *Phys. Rev. Lett.* **86**, 2653 (2001).
- ¹³ M.J. Graf and A.V. Balatsky, *Phys. Rev. B* **62**, 9697 (2000).
- ¹⁴ H. Tou, Y. Kitaoka, K. Ishida, K. Asayama, N. Kimura, Y. Onuki, E. Yamamoto, and K. Maezawa, *Phys. Rev. Lett.*, **77**, 1374 (1996).
- ¹⁵ K. Ishida, H. Mikuda, Y. Kitaoka, K. Asayama, Z.Q. Mao, Y. Mori, and Y. Maeno, *Nature*, **396**, 658 (1998).
- ¹⁶ R. Mahmoodi, S.N. Shevchenko and Yu.A. Kolesnichenko, *Fiz. Nizk. Temp.* **28**, 262(2002) [*Sov. J. Low Temp. Phys.* **28**, 184 (2002)].
- ¹⁷ N. Stefanakis, *Phys. Rev. B* **65**, 064533 (2002).
- ¹⁸ T. Yamashita, S. Takahashi, H. Imamura, and S. Maekawa, *Phys. Rev. B* **65**, 172509 (2002).
- ¹⁹ N.M. Chtchelkatchev *JETP Lett.* **78**, 230(2003) [*Pis'ma Zh. Eksp. Teor. Fiz.* **78**, 265 (2003)].
- ²⁰ G. Sun, Z. Zheng, D. Y. Xing, J. Dong and Z. D. Wang, *J. Phys.:Condens. Matter* **13**, 627 (2001).
- ²¹ S. Maekawa, J. Magn. Magn. Mater. **272**, 1459 (2004).
- ²² I.O. Kulik and A.N. Omelyanchouk, *Fiz. Nizk. Temp.*, **4**, 296 (1978) [*Sov. J. Low Temp. Phys.*, **4**, 142 (1978)].
- ²³ G. Rashedi and Yu.A. Kolesnichenko, *Phys. Rev. B* **69**, 024516 (2004).
- ²⁴ M.H.S. Amin, M.Coury, S.N. Rashkeev, A.N. Omelyanchouk, and A.M. Zagorskii, *Physica B*, **318**, 162 (2002).
- ²⁵ Yu. S. Barash, A.M. Bobkov, and M. Fogelström, *Phys. Rev. B* **64**, 214503 (2001).
- ²⁶ J.K. Viljas, E.V. Thuneberg, *Phys. Rev. B* **65**, 64530 (2002); J. Viljas, cond-mat/0004246.
- ²⁷ Z. Farai and M. Zareyan, *Phys. Rev. B* **69**, 014508 (2004).
- ²⁸ M. Freamat, K.-W. Ng, *Phys. Rev. B* **68**, 060507 (2003).
- ²⁹ S.-K. Yip, *Phys. Rev. Lett.* **83**, 3864 (1999).
- ³⁰ S. Backhaus, S. Pereverzev, R.W. Simmonds, A. Loshak, J.C. Davis, and R.E. Packard, *Nature* **392**, 687 (1998).
- ³¹ G. Eilenberger, *Z. Phys.*, **214**, 195 (1968).